How Universal Is the Coupling in the Sigma Model?

Denis Parganlija[*], Francesco Giacosa[*] and Dirk H. Rischke[†]

[*]*Institut für Theoretische Physik, Johann Wolfgang Goethe-Universität, Max von Laue-Str. 1, D-60438 Frankfurt am Main, Germany*
[†]*Institut für Theoretische Physik and Frankfurt Institut for Advanced Studies, Johann Wolfgang Goethe-Universität, Max von Laue-Str. 1, D-60438 Frankfurt am Main, Germany*

Abstract. We calculate pion-pion scattering lengths and sigma, rho and a_1 decay widths from a gauged linear sigma model with two flavours and its globally invariant generalisation.

INTRODUCTION

Quantum Chromodynamics (QCD) at low energies can be successfully described by means of effective approaches which display the same global symmetries of QCD, most notably the chiral $SU(N_f)_r \times SU(N_f)_l$ symmetry, where N_f is the number of flavors. Effective models are expressed in terms of hadronic degrees of freedom and not in terms of quarks and gluons. Spontaneous breaking of chiral symmetry implies that the pseudoscalar mesons (i.e., the pions) emerge as almost massless Goldstone bosons and the scalar states which are the corresponding chiral partners acquire a large mass. We distinguish between the nonlinear and linear realizations of chiral symmetry: while in the nonlinear case the scalar excitations are integrated out and only pseudoscalar mesons, interacting via derivative couplings, are left, in the linear case both scalar and pseudoscalar degrees of freedom are present.

In this paper we work with the latter by using a generalized linear sigma model in which, besides scalar and pseudoscalar mesons, also vector and axial-vector degrees of freedom are included. We first revisit the hypothesis of local chiral symmetry as described in Refs. [1, 2] and show that a successfull description of pion-pion scattering and some important decay widths cannot be achieved simultaneously in this framework. As outlined in Ref. [3], allowing for global chiral symmetry represents a viable extension: a first study in this direction is performed here in the case of $N_f = 2$.

The scalar fields entering the model are interpreted as quark-antiquark states in agreement with large-N_c counting rules. We then have two possible scenarios: (a) the resonances below 1 GeV $f_0(980)$, $a_0(980)$, $k(800)$ and $f_0(600)$ represent the quarkonia nonet. Thus, the states $f_0(600)$ and $a_0(980)$ are identified with the σ and the a_0 fields of our $N_f = 2$ model. (b) The quarkonia are heavier than 1 GeV: the resonances $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, $a_0(1450)$, $K_0(1430)$ describe a full nonet, in which the isoscalar states mix with the glueball [4]. In the case $N_f = 2$ the resonances $f_0(1370)$ and $a_0(1450)$ correspond to the σ and the a_0 fields. The scalars below 1 GeV, whose spectroscopic

wave functions possibly contain a dominant tetraquark or mesonic molecular contribution in this scenario [5, 6], may be introduced in the model as extra scalar fields. In this work we briefly outline how we intend to explore both scenarios (a) and (b) in the future.

THE MODEL AND ITS IMPLICATIONS

The gauged linear sigma model: The Lagrangian of the gauged linear sigma model with $U(2)_R \times U(2)_L$ symmetry reads [1]:

$$
\begin{aligned}
\mathscr{L} =\ & \mathrm{Tr}[(D^\mu \Phi)^\dagger (D^\mu \Phi)] - m_0^2 \, \mathrm{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\mathrm{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \, \mathrm{Tr}(\Phi^\dagger \Phi)^2 \\
& - \frac{1}{4} \mathrm{Tr}[(F_l^{\mu\nu})^2 + (F_r^{\mu\nu})^2] + \frac{m_1^2}{2} \mathrm{Tr}[(A_l^\mu)^2 + (A_r^\mu)^2] + \mathrm{Tr}[H(\Phi + \Phi^\dagger)] \\
& + c \, (\det \Phi + \det \Phi^\dagger) \,,
\end{aligned}
\tag{1}
$$

with $\Phi = (\sigma + i\eta)t^0 + (\vec{a}_0 + i\vec{\pi}) \cdot \vec{t}$ (scalar and pseudoscalar mesons), $A_{l,r}^\mu = (\omega^\mu \pm f_1^\mu)t^0 + (\vec{\rho}^\mu \pm \vec{a}_1^\mu) \cdot \vec{t}$ (vector and axialvector mesons), where $t^0, \vec{t}$ are the generators of $U(2)$; $D^\mu \Phi = \partial^\mu \Phi + ig(\Phi A_l^\mu - A_r^\mu \Phi)$ and $F_{l,r}^{\mu\nu} = \partial^\mu A_{l,r}^\nu - \partial^\nu A_{l,r}^\mu - ig[A_{l,r}^\mu, A_{l,r}^\nu]$. Explicit symmetry breaking is described by the term $\mathrm{Tr}[H(\Phi + \Phi^\dagger)] \equiv h\sigma (h = const.)$ and the chiral anomaly by the term $c \, (\det \Phi + \det \Phi^\dagger)$ [7]. Note that the local symmetry in the Lagrangian (1) is explicitly broken to a global symmetry by non-vanishing vector meson masses. The respective term gives rise to the celebrated current-field proportionality [1].

The explicit form of the Lagrangian after spontaneous symmetry breaking can be found, e.g., in Ref. [8]. Note that all the parameters in the Lagrangian are fixed by the tree-level meson masses and the pion decay constant f_π.

Scattering lengths: In order to calculate the scattering lengths, at tree level one has to compute the amplitude corresponding to the diagrams shown in Fig. 1.
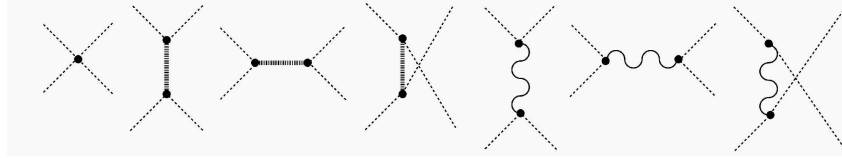

FIGURE 1. Tree-level pion-pion scattering (dashed line: pions; solid line: sigma; wavy line: rho mesons)

Partial wave decomposition [9] then leads to the following expressions for the scattering lengths in units of the pion mass:

$$
a_0^0 = \frac{m_\pi^2}{32\pi f_\pi^2} \left[7 + \frac{2}{Z^2} \frac{m_\pi^2}{m_\sigma^2} \left(1 - \frac{2}{Z^2}\right)^2 + \frac{3}{Z^2} \frac{m_\pi^2}{m_\sigma^2 - 4m_\pi^2} \left(1 + \frac{2}{Z^2}\right)^2 \right],
\tag{2}
$$

$$
a_0^2 = -\frac{m_\pi^2}{16\pi f_\pi^2} \left[1 - \frac{1}{Z^2} \frac{m_\pi^2}{m_\sigma^2} \left(1 - \frac{2}{Z^2}\right)^2 \right]
\tag{3}
$$

with $Z = \frac{m_{a_1}}{m_\rho}$ and $f_\pi = 92.4$ MeV. Note that the PDG value is $m_{a_1} = 1230$ MeV whereas the KSFR [10] rule suggests $m_{a_1} \stackrel{!}{=} \sqrt{2}m_\rho = 1097$ MeV.

Given the data on scattering lengths: $a_0^0 = 0.224 \pm 0.03$ and $a_0^2 = -0.037 \pm 0.024$ (NA48/2 Cusp) [11]; $a_0^0 = 0.233 \pm 0.023$ and $a_0^2 = -0.047 \pm 0.015$ (NA48/2 Ke4) [11], it follows from Eqs. (2) and (3) that within the framework of the Lagrangian (1) a light scalar meson with the mass $m_\sigma \simeq (315 - 345)$ MeV is favoured, depending on the choice of a_1 mass and thus the choice of the parameter Z.

Decay widths: The results for the decay widths of the sigma and the rho into two pions that follow from the Lagrangian (1) fail to reproduce experimental results [12]. The calculation of the $\sigma \to \pi\pi$ decay width yields

$$\Gamma_{\sigma \to \pi\pi} = \frac{3}{32\pi} \frac{m_\sigma^3}{Z^6 f_\pi^2} \sqrt{1 - \left(\frac{2m_\pi}{m_\sigma}\right)^2} \left[1 + \frac{m_\pi^2}{m_\sigma^2}(Z^2 - 2)\right]^2.$$

One obtains $\Gamma_{\sigma \to \pi\pi} < 107$ MeV for $m_\sigma < 800$ MeV, clearly too small when compared to the PDG value which ranges between 600 MeV and and 1.2 GeV.
The decay width of the ρ into two pions is given by

$$\Gamma_{\rho \to \pi\pi} = \frac{g^2}{192\pi} m_\rho \left[1 - \left(\frac{2m_\pi}{m_\rho}\right)^2\right]^{\frac{3}{2}} \left(1 + \frac{1}{Z^2}\right)^2; \, g = \frac{\sqrt{Z^2 - 1}}{Z} \frac{m_\rho}{f_\pi} \approx 6.51.$$

Then the value $\Gamma_{\rho \to \pi\pi} = 86.5$ MeV is obtained: almost a factor of two lower than the experimental value $(149.4. \pm 1.0)$MeV. Additionally, it also follows from Eq. (1) that $\Gamma_{a_1 \to \rho\pi} \approx 300$ MeV, with the experimental value at $(250 - 600)$MeV.

IMPROVEMENTS OF THE LAGRANGIAN

A possible solution to the problem of the decay widths mentioned in the previous section was discussed in Refs. [1, 2, 13] where terms of higher dimension have been added to the Lagrangian.
However, as in Ref. [3] we follow a different strategy: given that *i)* the local symmetry of the Lagrangian (1) is already broken to a global symmetry, and *ii)* there seems to be no reason why an effective field theory should have a local chiral symmetry if the same symmetry in the underlying theory (QCD) is a global one [3], one may promote the local symmetry in the Lagrangian (1) to a global one.
For a global chiral symmetry, up to scaling dimension four the following additional terms appear in the Lagrangian (1): $\text{Tr}(\Phi^\dagger\Phi)\text{Tr}(|A_l^\mu|^2 + |A_r^\mu|^2)$, $\text{Tr}(\Phi A_{\mu l}\Phi^\dagger A_r^\mu)$ and $\text{Tr}(|\Phi A_r^\mu|^2 + |A_l^\mu \Phi|^2)$ [14, 15, 16]. The consequences for the pion scattering lengths and decay widths arising from the globally invariant Lagrangian are under investigation [17]. In this work we will restrict our discussion to noticing that global invariance implies mathematically that the coupling constant in the covariant derivative $D^\mu \Phi = \partial^\mu \Phi + ig_1(\Phi A_l^\mu - A_r^\mu \Phi)$ no longer needs to be the same as the one appearing in the field strength tensor $F_{l,r}^{\mu\nu} = \partial^\mu A_{l,r}^\nu - \partial^\nu A_{l,r}^\mu - ig_2[A_{l,r}^\mu, A_{l,r}^\nu]$; the ensuing division of a single coupling constant (g) into two different ones means that the scalar-vector coupling (g_1) is no longer the same as the vector-vector coupling (g_2) and at the same

time it provides us with a new parameter needed to adjust the $\rho \to \pi\pi$ decay width to the experimentally very precisely (within ± 1.0 MeV) observed value.

A recalculation of the $\rho \to \pi\pi$ width then yields

$$\Gamma_{\rho \to \pi\pi} = \frac{m_\rho}{48\pi} \left[1 - \left(\frac{2m_\pi}{m_\rho} \right)^2 \right]^{\frac{3}{2}} \left[g_1 - \frac{g_2}{2} \left(1 - \frac{1}{Z^2} \right) \right]^2 .$$

Using the PDG value $\Gamma_{\rho \to \pi\pi} = 149.4$ MeV one obtains[1] $g_2 = 1.77$.

In the present model, the value of the $a_1 \to \rho\pi$ decay width depends on the value of the a_1 mass. Our results indicate that the a_1 resonance is broad (in accordance with the experiments); the exact value is $\Gamma_{a_1 \to \rho\pi} = 516$ MeV for $g_2 = 1.77$ and $m_{a_1} = 1097$ MeV (KSFR rule), and around 1.3 GeV for $m_{a_1} = 1230$ MeV. Improving the very large decay width value of a_1 at $m_{a_1} = 1230$ MeV is currently under investigation [17].

The value of the $\sigma \to \pi\pi$ decay width at threshold is not affected by the introduction of the new coupling constant g_2, hence it is still too small. To correct this, one or both of the following two steps may be taken:

- Investigating all globally symmetric terms with no new states added [17]; the σ field is then interpreted as $f_0(600)$ and a_0 field as $a_0(980)$ and both are treated as quarkonia.
- Assuming that the masses of scalar quarkonia are above 1 GeV, re-interpreting the σ field as $f_0(1370)$ and a_0 field as $a_0(1450)$ and adding new terms corresponding to $f_0(600)$ and $a_0(980)$ (similar to the work in Refs. [6]).

In this work we have taken the latter step and as a first solution to the σ decay width problem we have added a scalar-pion interaction term $\mathcal{L}_{\tilde{g},S} = \tilde{g} f_\pi S \left(\frac{\partial_\mu \vec{\pi}}{f_\pi} \right)^2$, where S is now interpreted as $f_0(600)$, to the Lagrangian of Eq. (1). Therefore, two new parameters are introduced ($\tilde{g}$ and m_S) that may be adjusted to the two scattering lengths a_0^0 and a_0^2. Then the value of the pion-pion $f_0(600)$ decay width may be calculated. A recalculation of the scattering lengths yields

$$a_0^{0new} = a_0^0 + \frac{\tilde{g}^2 m_\pi^4}{\pi} \left(\frac{1}{m_S^2} - \frac{3}{2} \frac{1}{4m_\pi^2 - m_S^2} \right), \quad a_0^{2new} = a_0^2 + \frac{\tilde{g}^2}{\pi} \frac{m_\pi^4}{m_S^2},$$

with a_0^0 and a_0^2 from (2) and (3) respectively. Our best values for the $f_0(600)$ mass and decay width are at $m_S = 608$ MeV and $\Gamma_{S \to \pi\pi} = 466$ MeV (obtained for $a_0^0 = 0.206$, $a_0^2 = -0.0295$ and $g_3 = 0.645$) which is within experimental values as quoted by the PDG [12]. This also means, however, that the quoted scattering lengths are within

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[1] There also is a second solution $g_2 = 41.48$, which, however, leads to an unphysical (i.e., too large) value of the a_1 decay width into rho and pion.

NA48/2 Cusp data and outside of NA48/2 Ke4 data [11]. This issue will be addressed in the future [17].

CONCLUSIONS AND OUTLOOK

A linear sigma model with vector and axial-vector mesons has been utilized to study important processes of low-energy QCD. The necessity and the consequences of abandoning local chiral symmetry and considering globally symmetric terms have been discussed. In the future, a complete study of globally symmetric terms up to fourth order and their influence on experimental results is required. In this way, we plan to address relevant issues concerning vacuum phenomenology, such as the nature of scalar mesons and the inclusion of the nucleon field together with its chiral partner [18]. Moreover, we plan to extend the work of Ref. [8] in order to consider chiral symmetry restoration at nonzero temperature.

ACKNOWLEDGMENTS

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